

Chapter 3

Section 3.7 – Implicit Differentiation

Example: $x^4 + y^4 = 16$

a. Find $\frac{dy}{dx}$

$$\frac{d}{dx}(x^4 + y^4) = 16$$

$$4x^3 + 4y^3 \frac{dy}{dx} = 0$$

$$4y^3 \frac{dy}{dx} = -4x^3$$

$$\boxed{\frac{dy}{dx} = \frac{-x^3}{y^3}}$$

b. Find $\frac{d^2 y}{dx^2}$

$$\frac{d^2 y}{dx^2} = \frac{y^3(-3x^2) - (-x^3)(3y^2)\frac{dy}{dx}}{y^6}$$

$$= \frac{-3x^2y^3 + 3x^3y^2\left(\frac{-x^3}{y^3}\right)}{y^6}$$



Example: $\frac{d}{dx}(y^5 + 3x^2y^2 + 5x^4 = 12)$

PRODUCT RULE

$$5y^4 \frac{dy}{dx} + 6xy^2 + 6x^2y \frac{dy}{dx} + 20x^3 = 0$$

$$5y^4 \frac{dy}{dx} + 6x^2y \frac{dy}{dx} = -6xy^2 - 20x^3$$

$$\frac{dy}{dx}(5y^4 + 6x^2y) = -6xy^2 - 20x^3$$

$$\boxed{\frac{dy}{dx} = \frac{-6xy^2 - 20x^3}{5y^4 + 6x^2y}}$$



Find the equation of the tangent line at $x = 1$ if $y^2 = 2x^3 - x^4$

$$\frac{d}{dx}(y^2 = 2x^3 - x^4)$$

$$2y \frac{dy}{dx} = 6x^2 - 4x^3$$

$$\frac{dy}{dx} = \frac{6x^2 - 4x^3}{2y}$$

$$\frac{dy}{dx} = \frac{3x^2 - 2x^3}{y}$$

y

Point: $\bar{y} = 2(1)^3 - (1)^4 = 1 \Rightarrow y = \pm 1$

\Rightarrow TWO POINTS $(1, 1)$ AND $(1, -1)$

AT $(1, 1)$:

$$m = \left. \frac{dy}{dx} \right|_{(1,1)} = \frac{1}{1} = 1$$

AT $(1, -1)$:

$$m = \left. \frac{dy}{dx} \right|_{(1,-1)} = \frac{1}{-1}$$

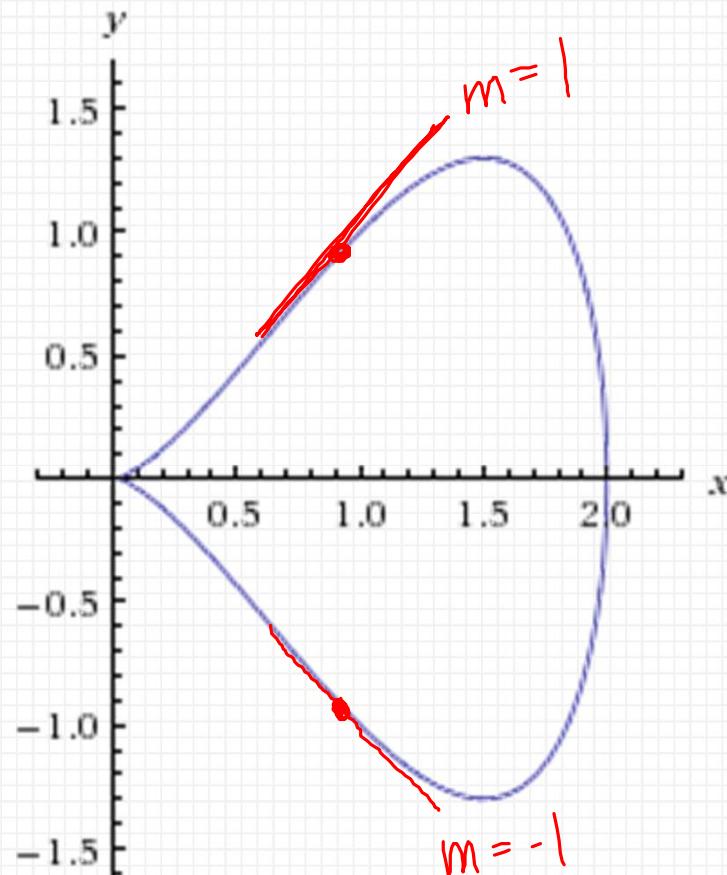
$$y - 1 = 1(x - 1)$$

$$y + 1 = -1(x - 1)$$



Find the equation of the tangent line at $x = 1$ if $y^2 = 2x^3 - x^4$

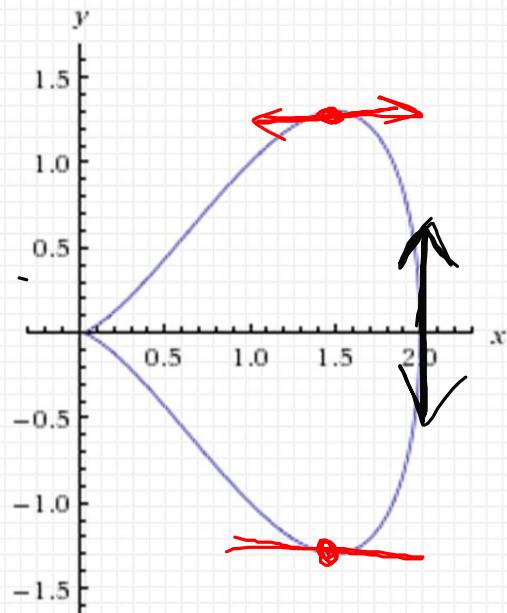
Graph:



Find the points where tangents lines are vertical or horizontal.

$$y^2 = 2x^3 - x^4$$

Graph:



HORIZONTAL : $m = 0$

$$\frac{dy}{dx} = 0 = \frac{3x^2 - 2x^3}{y} = 0 = x^2(3 - 2x) \Rightarrow x = 0, \frac{3}{2}$$

$$x = 0 \Rightarrow y^2 = 0 \Rightarrow y = 0 \Rightarrow (0, 0)$$

$$x = \frac{3}{2} \Rightarrow y^2 = 2\left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^4 \Rightarrow y = \pm 1.299$$

$$\left(\frac{3}{2}, 1.299\right), \left(\frac{3}{2}, -1.299\right)$$

VERTICAL: $\frac{dy}{dx}$ DNE

$$\Rightarrow y = 0 \Rightarrow 0 = 2x^3 - x^4 \\ 0 = x^3(2-x) \\ x = 0, 2$$

$$(0, 0) \Rightarrow \frac{dy}{dx} = \frac{0}{0} \quad ?$$

$$(2, 0)$$

MARIB

$$\frac{dy}{dx} = \frac{0}{0} \quad ?$$

?

?



Logarithmic Differentiation:

Example: $y = x^x$

$$y = f(x) \quad g(x)$$

$$\ln y = \ln x^x$$

$$\frac{d}{dx} (\ln y = x \ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$$

$$\frac{dy}{dx} = y (1 + \ln x) = \boxed{x^x (1 + \ln x)}$$

1. Take the natural log of both sides.
2. Simplify using properties of logs.
3. Differentiate implicitly.
4. Solve for y'
5. Plug in expression for y .



Logarithmic Differentiation:

Example: $y = (\sin x)^{\cos x}$

$$\frac{d}{dx} (\ln y = \cos x \ln(\sin x))$$

$$\frac{1}{y} \frac{dy}{dx} = \cos x \cdot \frac{\cos x}{\sin x} + \ln(\sin x) (-\sin x)$$

$$\frac{dy}{dx} = (\sin x)^{\cos x} \left(\frac{\cos^2 x}{\sin x} - \sin x \ln(\sin x) \right)$$



Logarithmic Differentiation:

Example: $y = \sqrt[3]{\frac{(x+1)^5(x+2)^2}{\sin^3 x}}$ 1/3

$$\ln y = \frac{1}{3} \ln \frac{(x+1)^5(x+2)^2}{\sin^3 x}$$

$$\ln y = \frac{1}{3} \left[5 \ln(x+1) + 2 \ln(x+2) - 3 \ln(\sin x) \right]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{3} \left[\frac{5}{x+1} + \frac{2}{x+2} - \frac{3 \cot x}{\sin x} \right] \Rightarrow \frac{dy}{dx} = \frac{1}{3} \sqrt[3]{\frac{(x+1)^5(x+2)^2}{\sin^3 x}} \left[\frac{5}{x+1} + \frac{2}{x+2} - 3 \cot x \right]$$



Homework/Classwork:

AP Packet: # 49 – 73 odd

